Math 10A with Professor Stankova
Quiz 3; Wednesday, 9/13/2017
Section #106; Time: 10 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** Let the domain of $f(x)$ be $[-1, 3]$. Then the domain of $f(2x+3)$ is $[2(-1)+3, 2(3)+3] = [1, 9]$.

2. True **FALSE** It is possible for a function to be differentiable but not continuous.

Show your work and justify your answers.

3. (10 points) Let $f(x) = x^3 \exp(-1/x^2)$ and $g(x) = f^{-1}(x)$ be the inverse of f.

   (a) (1 point) What is the domain of f?

   Solution: The only restriction is that $x \neq 0$ so $D = \mathbb{R}\backslash\{0\}$.

   (b) (1 point) Find $\lim_{x\to 0} f(x)$.

   Solution: Letting $x \to 0$, we have that $\frac{-1}{x^2} \to -\infty$ and $e^{-\infty} = 0$. So "plugging in 0" gives $\lim_{x\to 0} x^3 \exp(-1/x^2) = 0 \cdot 0 = 0$.

   (c) (5 points) Find $f'(x)$.

   Solution: First using the product rule then chain rule, we have that

$$\frac{d}{dx}(x^3 e^{-1/x^2}) = 3x^2 e^{-1/x^2} + x^3 e^{-1/x^2} \cdot \frac{2}{x^3} = e^{-1/x^2}(3x^2 + 2).$$

   (d) (3 points) Given that $f(1) = 1/e$, find $g'(1/e)$.

   Solution: Since $f(1) = 1/e$, we know that $g(1/e) = 1$. Then using the formula that $g'(x) = \frac{1}{f'(g(x))}$, we have that

$$g'(1/e) = \frac{1}{f'(g(1/e))} = \frac{1}{f'(1)} = \frac{1}{(1/e)(5)} = \frac{e}{5}.$$